Chapter 8

Factor analysis and PCA

8.1 Factor analysis

The history of the use of factor analysis in archaeology is recounted in Baxter (1994a, 2003). A summary is that 'factor analysis' was widely used (or abused) in applications to the mid-1980s, but has not been prominent since. It was widely confused with PCA and confusion still exists. The need for a separate chapter on factor analysis is questionable, but it has been 'promoted' in the fairly recent undergraduate text of VanPool and Leonard (2010) – at the apparent expense of PCA, though this may not be intentional – who perpetuate some of the misconceptions surrounding the distinction between the methods. This, with a summary of the historical background, is discussed in Section 8.4.

The view almost invariably expressed in texts written by statisticians, and adopted here, is that PCA and factor analysis are *different* methods and that maintaining this distinction is important. The view expressed in some archaeological writing (e.g., Drennan, 2009: 299–300) that the methods are conceptually different but that this is of no practical importance is only tenable if, having accepted there is a conceptual distinction, the consequences are then ignored. The concluding section returns to this argument.

Section 8.2 summarizes some of the main differences between PCA and factor analysis and 'problematic' aspects of application – that is, how the methods differ and why. The example in Section 8.3.1 illustrates the effects of *rotation* and the choice of coefficient constraints on the outcome of a PCA. The idea of rotation is 'borrowed' from factor analysis where it is fundamental. The intent is to take an initial 'solution' and modify it in the interests of 'interpretability'. This can be done in many different ways, so there is an unavoidable 'indeterminacy' involved in any application of factor analysis. The example in Section 8.3.2 shows how numerical results can vary as a consequence of the choices that have to be made in implemention. This is, regrettably, a subject where understanding of the mathematics that underpins the methods helps to clarify why they are different, and a succinct account is provided in Appendix D.

8.2 Theory - a brief summary

Remember that PCs are constructed as linear combinations of Y_i

$$Z_j = a_{j1}Y_1 + a_{j2}Y_2 + \ldots + a_{jp}Y_p \tag{8.1}$$

that are uncorrelated and, subject to this, account for successively decreasing amounts of the variance in the data. Determining the a_{ji} is a purely mathematical operation that depends, additionally, on constraints that it is necessary to impose for a unique solution (constraint 7.1 or 7.2)[1].

Constraint (7.1) is used in `R` and other software, where PCA and factor analysis are clearly distinguished. An important exception, and a source of confusion, is the widely-used `SPSS` package, where PCA is treated as a particular case of factor analysis and constraint (7.2) is used.

Equation (8.1), subject to the chosen constraint, has a unique solution determined mathematically. The relationship can be 'inverted' to obtain an expression for the variables as a function of the components (Section D.3.1).

$$Y_j = a_{1j}Z_1 + a_{2j}Z_2 + \ldots + a_{pj}Z_p \tag{8.2}$$

This does not involves any notion of random variation such as might be represented by an 'error' term, and does not require estimation of the coefficients with associated measures of uncertainty[2].

In contrast, and it is important, factor analysis requires that a statistical model be specified for the data. This has the form

$$Y_j = b_{1j}F_1 + b_{2j}F_2 + \ldots + b_{qj}F_q + \varepsilon_j \tag{8.3}$$

where the final term is a random component ('error' term). The *loadings* b_{ij} that determine the factors must be estimated. In PCA there are p components, of which a subset, q, may be selected for presenting results; in factor analysis there are $q < p$ factors. The hope is that these *latent variables* can be assigned a 'meaning' at the interpretive stage as unobservable variables that explain the observed covariance

[1] A distinction, not always made in the literature, will be maintained between the use of the term *coefficients* for PCA and *loadings* for factor analysis. Similarly, *component* and *factor* rotation are distinguished.

[2] To be clear about this, the way in which component coefficients are determined using mathematical methods is being distinguished from statistical estimation.

structure of the data. In PCA no such 'meaning' is necessarily attributed to the components, nor need there be for productive analysis.

The covariance matrix of the data is **S**. In PCA the coefficients are extracted via the singular value decomposition (SVD) of **S**, or some procedure with equivalent effect. This is a mathematical operation that produces PCs with the required properties. In factor analysis **S** is broken down into the sum of two components which are the contributions of the random terms and the systematic components represented by the factors as in equation (D.6). A distinction often made is that, in contrast to PCA where the aim is to account for as much variance as possible with a small number of components, the emphasis in factor analysis is on the covariance structure of the data rather than the variances, with the effort directed at modeling this in terms of *common factors* that explain this structure.

Another way of stating this is that PCA and factor analysis have different aims. Factor analysis implies a belief that unobservable variables – with a 'meaning' that can be articulated – explain the covariance structure of the data. Factor analyis results are unavoidably affected by analytical choices for which definitive statistical theoretical guidance does not exist. Some commentators are uneasy about the 'flexibility' of interpretation this allows; what is not in doubt is the 'indeterminacy' in the results (i.e. factors identified) that can be obtained. Factor *rotation* is at the heart of this.

Conditionally on the data pre-treatment used, PCA provides a unique solution to the problem it is designed for. Any attempt to modify the PCA solution destroys its optimality properties. Not so factor analysis, since it does not attempt to optimize any well defined criterion that the factors should satisfy. An initial solution (i.e. determination of the b_{ij}) is not unique. Rotation has the aim of achieving simple and interpretable structure. The idea is that it is easier to attach a 'meaningful' label to rotated factors than it is to the initial solution[3].

To summarize, in factor analysis factor rotation is *de riguer*. Formally, in PCA, components can be rotated but the results then do not have the optimal properties PCA is designed to achieve. The witting use of rotation of PCs to enhance interpretation is sometimes seen; the confusion of rotated PCA with factor analysis is more pernicious. This is discussed further, in the context of the archaeological literature, in Section 8.4. Apart from the choice of a rotation method (Section D.3.2), many methods of factor extraction are possible, contributing to the variety of solutions possible. Section D.3.3 describes some of these and Section 8.3.2 provides illustrative applications.

[3]It seems to be implicit, in this approach, that factors should have simple structure, with loadings either 'high' or 'close' to zero. It is not obvious that there is a logical reason why latent variables should have simple structure, so the requirement is really one of interpretive convenience.

8.3 Examples

8.3.1 PCA and rotation

The data used for illustration are the stone axe data of Tables B.9 to B.11 for butt types 1, 3 and 5, already analyzed in some detail in Section 7.4. The sequence of commands that follows provides the basis for the results given in the upper-part of Table 8.1, and uses the constraint in equation (7.1).

```
PC1 <- prcomp(SAst)
PC1L <- PC1$rotation[, 1:4]
PC1R <- varimax(PC1L)$loadings
```

	Components				Components			
	1	2	3	4	1	2	3	4
	PCA (`prcomp`)				Varimax rotation (cutoff = 0.3)			
L1	-0.33	0.18	0.16	0.09	-0.39			
L2	-0.23	0.45	0.53	0.40	-0.80			
B1	-0.33	-0.29	-0.04	0.21		-0.49		
B2	-0.31	-0.23	-0.02	0.37		-0.49		
B3	-0.32	-0.31	-0.08	0.20		-0.50		
WC	-0.29	-0.44	-0.13	0.06		-0.51		
DC	-0.23	-0.32	0.58	-0.67				-0.96
TH	-0.32	0.26	-0.29	-0.24			-0.55	
L3	-0.30	0.22	0.22	-0.04	-0.39			
T1	-0.32	0.25	-0.33	-0.23			-0.57	
T2	-0.32	0.24	-0.29	-0.21			-0.54	
	PCA (`principal`)				Varimax rotation (cutoff = 0.45)			
L1	0.91	0.23	0.12	-0.06	0.60		0.56	
L2	0.64	0.57	0.38	-0.28			0.89	
B1	0.91	-0.37	-0.03	-0.15		-0.88		
B2	0.86	-0.29	-0.01	-0.26		-0.83		
B3	0.90	-0.39	-0.06	-0.14		-0.89		
WC	0.81	-0.56	-0.09	-0.04		-0.91		
DC	0.65	-0.41	0.41	0.47				0.86
TH	0.89	0.33	-0.21	0.17	0.88			
L3	0.85	0.28	0.16	0.03	0.61		0.55	
T1	0.88	0.32	-0.24	0.16	0.88			
T2	0.90	0.30	-0.21	0.15	0.87			

Table 8.1: *Coefficients and varimax rotated PCs for different treatments of the stone axe data of Tables B.9 to B.11.*

Here, `PC1L <- PC1$rotation[, 1:4]` extracts the coefficients for the first four components. These are shown in the upper-left of Table 8.1 and, apart from rounding, are the same as those in Table 7.3. The eigenvalues for the first two PCs are 2.79 and 1.27; for the third and fourth PCs they are 0.72 and 0.70. The first two components account separately for 71.0% and 14.6% of the total variance, and cumulatively for 85.6%; the next two account for about 9% cumulatively so about 95% of the variance is attributable to the first four components (Table 7.4).

The interpretation in terms of size and shape components is discussed in Section 7.4.2. While several criteria for component selection in advance of rotation, such as Kaiser's rule, would lead to a choice of two, more forgiving criteria would lead to different choices (e.g., a modified Kaiser's rule using 0.7, or what VanPool and Leonard (2010: Chapter 15) state is a 'common cutoff' for the cumulative percentage used to select the number of components of 95%). Four components will be rotated for illustrative purposes.

Jolliffe's (2002, pp. 112–133) account of component selection in PCA concludes that rules having a 'sound statistical foundation' seem 'to offer little advantage over the simpler methods in most cirumstances'. Since he also notes that the simpler methods are 'very much ad-hoc rules-of-thumb' (page 112), and that the choice of the number of components to rotate 'can have a large effect on the results after rotation' (page 271) this would appear to leave the aspirant rotator of principle components in something of a quandary when deciding what to do!

The command `PC1R <- varimax(PC1L)$loadings` uses the `varimax` function to rotate the four components with results shown in the upper-right table. The common convention of suppressing rotated coefficient values below some cutoff, is followed. The default is to suppress values for which $|a_{ij}| < 0.1$. Here,

```
print(PC1R, digits = 2, cutoff = 0.3).
```

which rounds the a_{ij} to two digits and prints those for which $a_{ij} > 0.3$, is used, as in the table to the upper-right. The cutoff, 0.3, is arbitrary (as is the default) but designed to emphasize the most important clusters of variables that characterize each rotated component.

How does interpretation differ from the unrotated solution, if at all? The plots in Figure 7.5, based on the first three components, can be clearly interpreted in terms of three clusters of variables corresponding to 'length', 'breadth' and 'thickness' with depth of cutting-edge as an isolated variable. The rotated solution to the upper-right of Table 8.1 does not add to this, but loses the direct interpretation in terms of 'size' and 'shape' components evident in the unrotated analysis. We might, if the fancy takes us, call these variable clusters 'factors', but the analysis is not a factor analysis.

The lower set of tables repeats the analysis obtained with the `principal` function from the `psych` package using constraint (7.2). The number of components (called 'factors') to extract needs to be specified explicitly and varimax rotation is applied by default. To get a rotated PCA the following can be used.

```
library(psych)
PC2 <- principal(SAst, nfactors = 11, rotate = "none")
```

```
PC2L <- PC2$loadings[, 1:4]
PC2R <- varimax(PC2L)$loadings
```

The argument `nfactors` specifies the number of components/factors to extract – the maximum of 11 in this case – and the `rotate = "none"` argument suppresses rotation. Thereafter things proceed as previously. Because of the different constraints, coefficients in the two unrotated analyses differ by a constant factor. The outcome of rotation depends on the constraint used (Jolliffe, 2002: 272–74) but both lose the variance-maximization properties of the unrotated solution and the property that component scores are uncorrelated.

After rotation coefficients no longer differ by a constant factor. The results in the lower part of Table 8.1 for `principal` can be compared with those from `prcomp`. A cutoff of 0.3 was used for the varimax rotated components in the latter case as it divided the variables neatly into different types; not such a neat division was possible for the results from `principal`, a cutoff of 0.45 eventually being chosen[4].

8.3.2 Variants of factor analysis

The data used are a 'classic' set of measurements on 30 La Tène Bronze Age fibulae from Münsingen, Switzerland. These were used in several early experimental studies of applications of multivariate methods in archaeology, in the late 1960s and early 1970s. They are published as Table 9.1 in Doran and Hodson (1975) and reproduced in Table B.15. There are three angular measurements and one variable of counts, with the other dimensions measured as millimeters. One of these variables has some values that could not be ascertained, and one fibula has missing data. Doran and Hodson replace these with estimates; here the offending row and column have been omitted in the analyses to follow so a 29×12 data matrix is used.

Doran and Hodson (1975: 225) stress that the data set, which is small, was intended to 'test out alternative methods and *not* to provide a useful archaeological classification' (their emphasis). This is the spirit in which the data are used here.

Other than the angular data, and following Doran and Hodson, variables are transformed to logarithms before analysis. There are some zero values; following Doran and Hodson 0.1 was added to all the non-angular data before taking

[4]With the `principal` function, rotated components do not necessarily retain the same ordering when compared to the original components they most resemble. Rotated components 2 and 4 in both analyses have a similar interpretation; component 1 in the `prcomp` rotation resembles the third rotated component for `principal`; and the third rotated component for `prcomp` and first component fro `principal` may be interpreted as 'thickness'. Broadly, though, the analyses lead to similar interpretations.

(natural) logarithms[5].

Following suggestions in Section D.4 for comparisons that might be of interest the upper part of Table 8.2 contrasts the results of orthogonal varimax rotation using principal axis and maximum-likelihood factor analysis; the lower part of the table provides a similar contrast using oblique oblimin rotation. The default in the `fa` function from the `psych` package was used for all analyses.

	Factors				Factors			
	A	B	C	D	A	B	C	D
	Principal axis factor analysis, varimax rotation				Maximum-likelihood, varimax rotation			
FL		0.83				0.81		
BH	0.59	-0.51		-0.45	0.58	-0.47		-0.50
CD	0.65				0.60			
ED	0.49		0.67		0.46		0.73	
FEL		0.88				0.94		
C	0.59	0.54		0.53	0.53	0.42	0.43	0.59
BW								
BT		-0.46	0.54	-0.43		-0.45	0.57	-0.59
Coils			0.68				0.48	
BFA	-0.74			0.40	-0.83			
FA				0.77				0.73
BRA	-0.85				-0.82			
	Principal axis factor analysis, oblimin rotation				Maximum-likelihood, oblimin rotation			
FL			0.50	0.54	0.41		0.51	0.54
BH	-0.59	0.43			-0.55	0.45		
CD	-0.44			-0.41				
ED		0.65				0.71	0.45	
FEL				0.91				0.93
C			1.01				0.99	
BW		0.45						
BT		0.69				0.88		
Coils	0.42	0.67				0.46		
BFA	0.84				0.93			
FA	0.44		0.44	-0.46		-0.41		-0.44
BRA	0.84				0.80			

Table 8.2: *Factor loadings from different analyses of the Bronze Age fibulae data of Table B.15, treated as described in the text and extracting four factors.*

As in Doran and Hodson (1975: 200–01), using Kaiser's rule, four factors were extracted. The code that follows is for principal axis factor analysis with varimax rotation.

```
FA <- fa(y, nfactors = 4, fm = "pa", rotate = "varimax")
```

where `y` is the data matrix determined as previously described.

[5] I have been unable to reproduce the results of this transformation given in Table 9.2 of Doran and Hodson, other than for the counted variable. My numbers don't differ too much from theirs, and I get similar results when reproducing their analyses.

The method selection argument `fm = "pa"` selects principal axis factoring, while `rotate = "varimax"` gives the rotation method to use. Change these to `fm = "ml"` and `rotate = "oblimin"` for maximum-likelihood estimation and oblimin rotation. There are other options available; `"oblimin"` is the default rotation and `"minres"` the default estimation method. The latter produces an ordinary least squares solution; the documention states it will produce results very similar to maximum-likelihood and it did so here (not shown).

The loadings were obtained with a cutoff of 0.4 for all the analyses reported, using the `loadings` function and the `print` method associated with it.

```
print(loadings(FA), digits = 2, cut = .4)
```

In Table 8.2 factors are ordered to make comparisons more readily between different analyses, rather than in the order that occurs in `fa` output.With relatively minor variations the two varimax analyses show fairly similar results to each other, and the two oblique rotations are also quite similar to each other.

The more obvious differences that can be seen, which do not depend on the cutoff used, arise in the comparison of the orthogonal and oblique rotations. Most obviously, perhaps, the catchplate dimension (C) does not stand out in any of the factors obtained using the varimax rotation, but dominates the third factor in the oblimin rotations. Similarly, factor D is dominated by the foot extension length (FEL) which has a high loading for factor B in the varimax rotations but does not dominate (or define) that factor. It is, in fact, difficult to discern much correspondence between the varimax and oblimin rotations. There is some similarity between the third factor for the former and the second factor for the latter.

For this example at least the results are sensitive to the choice of rotation method. The other potentially important source of difference concerns the choice of numbers of factors to rotate (Section D.4). For illustration Table 8.3 presents the results from a maximum-likelihood analysis using both varimax and oblimin rotation and extracting three factors. This may be compared with the output from Table 8.2.

For the varimax rotation factor C in Table 8.2 and factor B in Table 8.3 compare reasonably well; there is also some correspondence between factor A in the different analyses. Factor C in the three-factor analysis has no clear relationship to either factor B or D in the four-factor analysis or to any simple combination of them.

For the oblimin rotation Factor C in all analyses, which is dominated by the catchplate variable, corresponds well. The bow-angle variables are the most important in defining factor A in all analyses but the contributions of foot length (FL) and coil diameter (CD) make somewhat greater and non-trivial contributions in the three-factor analysis.

This leaves factor B in the three-factor analysis to be compared with factors B and D from the four-factor analyses. There is a quite good correspondence with

	Factor A	Factor B	Factor C	Factor A	Factor B	Factor C
	Maximum-likelihood, varimax			Maximum-likelihood, oblimin		
FL	-0.42		0.74	0.61		0.58
BH	0.72		-0.56	-0.66	0.44	
CD	0.70			-0.62		
ED	0.51	0.72			0.72	0.43
FEL	-0.47		0.44	0.45		
C	0.41		0.83			0.99
BW						
BT		0.62	-0.68		0.91	
Coils		0.50			0.50	
BFA	-0.83			0.87		
FA			0.50			
BRA	-0.84			0.85		

Table 8.3: *Factor loadings from maximum-likelihood factor analyses of the Bronze Age fibulae data of Table B.15, using varimax and oblimin rotation for three factors.*

factor B from the four-factor oblimin rotation; less so with the varimax rotation. Factor D in the four-factor analysis, largely determined by foot extension length, does not correspond to anything in the three-factor analysis.

The sole intention here has been to demonstrate that the choices to be made in conducting a factor analysis can have a non-trivial effect on the numerical output obtained. This Tables 8.2 and 8.3 do, particularly with respect to the rotation method used and numbers of factors rotated.

8.4 Factor analysis in archaeology

The importance of the paper by Binford and Binford (1966), which popularized the use of factor analysis in archaeology in the 1970s, is widely recognized (e.g., Doran and Hodson, 1975: 203–05; Orton, 1980: 136–39; Read, 1989) even when commentators disagree with aspects of it. Read (1989: 6–7) commented to the effect that the Binford's application of 'factor analysis', which was actually PCA with rotation, could serve 'as an example of incorrectly applied statistical methods'.

Vierra and Carlson (1981) listed over 70 studies between 1970 and 1978 that called their methodology 'factor analysis'. Their Table 1 gives details of 43 applications; more than half (27) were PCA, mainly with varimax rotation. Baxter (1994a: 277–79) notes about 20 additional analyses, mostly to the mid-1980s, that were dominated by PCA with varimax rotation. That is, more than 20 years after the introduction of factor analysis into the archaeological literature, confusion still existed between it and PCA.

There was a visible decline in archaeological uses of factor analysis and other multivariate methods from the mid-1980s or so arising from a backlash against the (mis)use of these methods. Baxter(1994a: 8) suggested that a consequence of this was that 'methodologically useful babies were unfairly thrown out with the theoretical bathwater'. Happily most of these babies survived to grow to maturity and become useful citizens in the world of quantitative archaeology; however, it was also suggested that factor analysis had 'possibly been lost with the bathwater, but the loss [was] not necessarily one to mourn' (Baxter, 1994a: 86).

Doran and Hodson (1975: 197–205) summarize their attitude towards uses of factor analysis up to that data as 'not very favourable'. Not all quantitatively able archaeologists have been so troubled. Cowgill (1977a), in his review of Doran and Hodson (1975), thought they exaggerated the distinction between PCA and factor analysis. An obvious comment is that if the techniques really are so similar, why bother with anything other than the simpler PCA methodology?

We return to this after first discussing VanPool and Leonard's (2010) treatment of the subject which, in my view, unwittingly encapaulates many of the reasons for the confusion between PCA and factor analysis in archaeological usage[6]. Put bluntly, I think they 'oversell' factor analysis, avoiding contentious issues that arise in using it. The distinction made between factor analysis and PCA is confused and the subject of what follows.

There is an absence of reference to texts that might be viewed as forerunners or 'competitors' (Doran and Hodson, 1975; Orton, 1980; Shennan, 1997; Drennan, 2009; Fletcher and Lock, 2005) so the reader is not exposed to more qualified assessments of factor analysis that have been voiced within some of these texts. This comment also applies to the wider journal literature. References to practical applications are very limited and hardly calculated to persuade the reader that factor analysis is a 'live' topic in archaeology.

The exception to this general lack of acknowledgment of a critical literature is Jolliffe (2002) who is cited in support of views expressed by the authors that arguably misrepresent what he says. It is stated that '[p]rincipal component and factor analysis are *very similar* but differ in the way they measure variation' (my emphasis) citing Jolliffe (2002: 180–96) as the authority for this. Jolliffe says no such thing; both PCA and factor analyses measure (co)variance in the same way, but the emphasis in PCA is on accounting for the variance (the diagonal elements of the covariance matrix) whereas factor analysis concentrates on modeling, or 'explaining', the covariance structure (off-diagonal elements) of the covariance matrix.

More worryingly, contrast the claim that the two methods are 'very similar' with what Jolliffe writes. To wit, 'the view [that PCA is a special case of factor analysis] is misguided since PCA and factor analysis, as usually defined, are really quite distinct techniques' (p. 150); 'a major distinction between factor analysis and PCA is that there is a definite model underlying factor analysis, but for most

[6] I'm working from the Kindle edition of the book, so can't give exact page references.

purposes no model is assumed in PCA' (p. 158); and 'there are many ways in which PCA and factor analysis differ from one another' (p. 160). This is not a ringing endorsement of the claim that the methods are 'very similar'.

VanPool and Leonard state that '[p]erhaps it isn't necessary to go so far as to say one should never use principal component analysis, but it is fair to say that it should only be used when the researcher can be reasonably sure that specific variance and error is small'. This follows an apparently supportive quote from Jolliffe. It can be read as asserting a preference for factor analysis over PCA and can charitably be described as misleading. To impose sense on it, it needs be interpreted as saying that *if* factor analysis is the appropriate method of analysis then PCA, as a surrogate for a proper factor analysis, should not be used.

This is important and worth spelling out in detail since it is at the heart of the confusion between factor analysis and PCA. The supposedly 'supportive' remark of Jolliffe is prefaced with another quote to the effect that '*various authors*' (my emphasis) have concluded that '... principal component analysis should not be used if a researcher wishes to obtain parameters reflecting latent constructs or factors'. This is from a single author, Widaman (1993), quoted exactly as above by Jolliffe (2002: 161). Widaman's remark was made in the context of an earlier 1990s disussion which, in Jolliffe's words, was underpinned by the 'assumption that unobservable factors are being sought from which the observed behavioural variables can be derived'. Jolliffe concludes that 'Factor analysis is clearly designed with this objective in mind, whereas PCA does not directly address it. Thus, at best, PCA provides an approximation to what is truly required'. Only this last sentence is referenced by VanPool and Leonard as a 'supportive quote'; without very careful reading and reference back to the original source it is all too easily understood as a fairly general view of Jolliffe, rather than specific comment on the view of a particular scholar made in the context of a focused 1990s debate in the behavioral sciences literature.

The fundamental problem with VanPool and Leonard's treatment is that it appears to be founded on the 'PCA as a special case of factor analysis' philosophy, with PCA then found wanting, rather than the 'PCA and factor analysis as distinct methods' philosophy. This perpetuates the crop of confusion between PCA and factor analysis sown by Binford and Binford (1966) that Read (1989) suggested was an exemplar of 'incorrectly applied' methodology.

Another problematic assertion is the statement that 'From the perspective of both techniques there are three "types" of variation in a data set; common, specific, and error ... Both [techiques] are excellent means of measuring common variance, but they differ in their treatment of specific variance and error.', followed by factor analysis 'only measures common variance' whereas PCA 'doesn't mathematically discard the specific variance and error as factor analysis does'. The PCA formulation does not involve any conception of specific or error variance; the statement only makes sense if PCA is treated as an inferior way of undertaking a factor analysis, rather than as a method in its own right.

This view that PCA and factor analysis are properly treated as distinct methods is overwhelmingly that of texts dealing with factor analysis and PCA writen by statisticians. Chatfield and Collins' (1980: 89) comment, in their introductory text on multivariate analysis, that 'we recommend that factor analysis should not be used in most practical situations' is at one extreme, but not untypical. Jolliffe (2002) provides a more dispassionate account. Other works in the same vein as Jolliffe's are listed in Section D.4. All are agreed that factor analysis and PCA are different methods; that the former involves a model for the data; and that *if* this model is appropriate PCA (with or without rotation) is not an optimal method for extracting the factors of interest. Claims that the two methods are 'very similar' or that they typically lead to very similar results (which can be queried[7]) fail to acknowledge what many scholars regard as fundamental differences between the methods.

One can take an entirely pragmatic view of this and ask how useful the methods have proved to be for archaeological data analysis. It would be straightforward to put together a book – albeit repetitive in places – consisting solely of uncontentious applications of PCA that produce readily understood results that have been found to be useful; such a book would be populated with examples from the mid-1960s to the present day. It would, I suspect, be a major challenge to do this for applications of factor analysis, avoiding studies where PCA masquerades as factor analysis. If the relative merits of the two methods must be discussed this might be seen as an acid test. Perhaps it isn't necessary to go so far as to say one should never use factor analysis (and indeed would be foolish to do so) but it is fair to say that it should only be 'promoted' if it has been demonstrably useful, and where the distinction between it and PCA is clearly maintained.

[7]The *raison dêtre* of factor analysis is that rotation, and the choice of method of rotation, *does* make a difference. It would seem to follow logically that claims to the effect that PCA and factor analysis lead to very similar results are only tenable if you envisage a PCA solution being subjected to the same rotational procedures as the factor analysis to which it is being compared. Even if it then turns out that claims about similarity are valid the thinking is predicated on the 'PCA as a special case of factor analysis' philosophy.